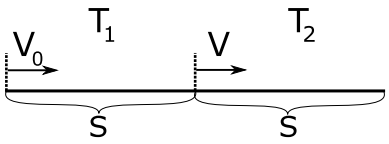
Олимпиада «Phystech International» - 2016. Физика. Решения. 09-01

1. a<0; $V=V_0=aT_1$; $V_0=V-aT_1$ (1)



$$S = V_0 T_1 + \frac{aT_1^2}{2} \quad (2)$$

(1)→(2): $S = VT_1 - aT_1^2 + \frac{aT_1^2}{2}$;

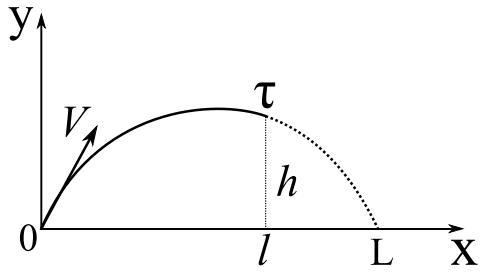
$$S = VT_1 - \frac{aT_1^2}{2} \Rightarrow a = \frac{2(VT_1 - S)}{T_1^2};$$

$$S = VT_2 + \frac{aT_2^2}{2} = VT_2 + \frac{(VT_1 - S)T_2^2}{T_1^2};$$

$$V = \frac{S(T_1^2 + T_2^2)}{T_1 T_2 (T_1 + T_2)} = \frac{24 \cdot (16+36)}{4 \cdot 6 \cdot 10} = 5{,}2 \quad \text{м/с}$$

2. $h = V_y\tau - \frac{g\tau^2}{2} \Rightarrow V_y = \frac{h}{\tau} + \frac{g\tau}{2}$

Время полёта: $T = \frac{2V_y}{g} = \frac{2h}{g\tau} + \tau$



$$l = V_x\tau \Rightarrow V_x = \frac{l}{\tau}; L = V_x T = \frac{l}{\tau} \cdot \left[\frac{2h}{g\tau} + \tau\right] = \frac{2lh}{g\tau^2} + l = l\left(1 + \frac{2h}{g\tau^2}\right) =$$

$$= 866 \cdot \left(1 + \frac{2 \cdot 375}{10 \cdot 25}\right) = 3464 м$$

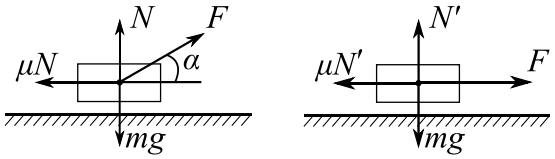
3. $N + F\sin\alpha = mg;\ N = mg - F\sin\alpha$ (1)

$F\cos\alpha - \mu N = F - \mu mg$ (2)

(1)→(2):

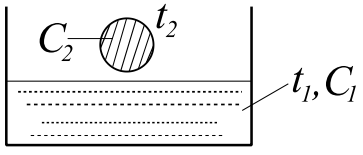
$F\cos\alpha - \mu mg + \mu F\sin\alpha = F - \mu mg$;

$\cos\alpha + \mu\sin\alpha = 1;$



$$\mu = \frac{1-\cos\alpha}{\sin\alpha} = \frac{1-\frac{1}{2}}{\sqrt{3}/2} = \frac{1}{\sqrt{3}} \approx 0{,}58$$

4. $\left.\begin{aligned} C_1(t_3 - t_1) &= C_2(t_2 - t_3) \\ C_1(t - t_1) &= 2C_2(t_2 - t) \end{aligned}\right\} \Rightarrow \frac{t_3 - t_1}{t - t_1} = \frac{t_2 - t_3}{2(t_2 - t)}$

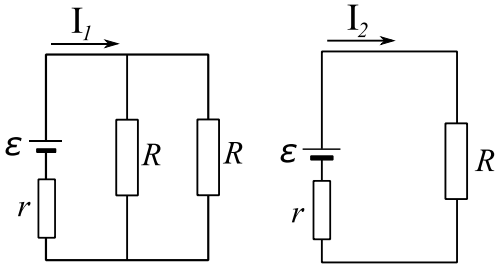


$$t=\frac{t_3(2t_2-t_1)-t_1t_2}{t_2+t_3-2t_1}=\frac{40\cdot(2\cdot100-10)-10\cdot100}{100+40-2\cdot10}=\frac{40\cdot190-1000}{120}=55^0\,\text{C}$$

5.

1) $\varepsilon=I_1\left(r+\frac{R}{2}\right)$ (1)

$$Q=I_1^2\cdot\frac{R}{2}=\frac{\varepsilon^2}{\left(r+\frac{R}{2}\right)^2}\frac{R}{2} \quad (2)$$



2) $\varepsilon=I_2(r+R)$ (3)

$$Q=I_2^2R=\frac{\varepsilon^2}{(r+R)^2}R \quad (4)$$

Из (2), (4): $\frac{\varepsilon^2}{\left(r+\frac{R}{2}\right)^2}\cdot\frac{R}{2}=\frac{\varepsilon^2}{(r+R)^2}\cdot R$

$$\left(r+\frac{R}{2}\right)\sqrt{2}=r+R;\quad r\left(\sqrt{2}-1\right)=R\left(1-\frac{1}{\sqrt{2}}\right);\qquad r=\frac{R}{\sqrt{2}}=\frac{28}{\sqrt{2}}\approx19{,}8\ \text{Ом}$$

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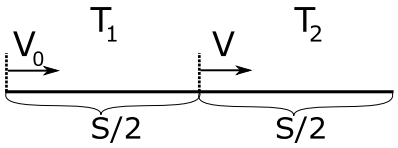
1. A>0; $V=V_0=aT_1$; $V_0=V-aT_1$ (1)

$$\frac{S}{2}=V_0T_1+\frac{aT_1^2}{2} \quad (2)$$

(1)→(2): $\frac{S}{2}=VT_1-aT_1^2+\frac{aT_1^2}{2}$;



$$\frac{S}{2}=VT_1-\frac{aT_1^2}{2}\Rightarrow a=\frac{2VT_1-S}{T_1^2};$$

$$\frac{S}{2}=VT_2+\frac{aT_2^2}{2}=VT_2+\frac{\left(VT_1-\frac{S}{2}\right)T_2^2}{T_2^2};$$

$$S=2\frac{V(T_1+T_2)T_1T_2}{T_1^2+T_2^2}=2\frac{5{,}2\cdot(6+4)\cdot6\cdot4}{6^2+4^2}=48\text{ м}.$$

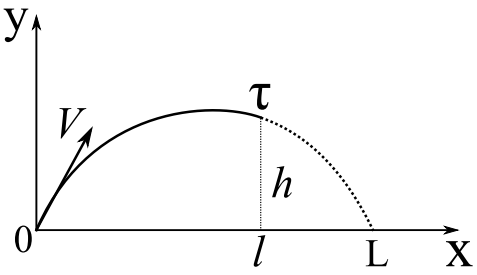
2. $h=V_y\tau-\frac{g\tau^2}{2}\Rightarrow V_y=\frac{h}{\tau}+\frac{g\tau}{2}$

Время полёта: $T=\frac{2V_y}{g}=\frac{2h}{g\tau}+\tau$

$l=V_x\tau\Rightarrow V_x=\frac{l}{\tau}$

$L=V_xT=\frac{l}{\tau}\left[\frac{2h}{g\tau}+\tau\right]=\frac{2lh}{g\tau^2}+l$

$S=L-l=\frac{2lh}{g\tau^2}=\frac{2\cdot2600\cdot375}{10\cdot15^2}\approx867\text{ м}.$



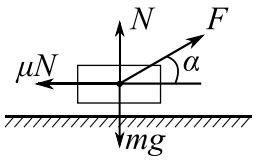
3. $N+F\sin\alpha=mg;\ N=mg-F\sin\alpha$ (1)

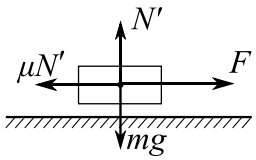
$F\cos\alpha-\mu N=F-\mu mg$ (2)

(1)→(2): $F\cos\alpha-\mu mg+\mu F\sin\alpha=F-\mu mg$; $\cos\alpha+\mu\sin\alpha=1$;

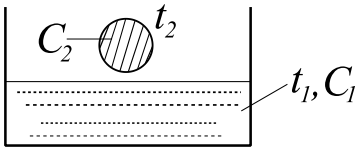
$$\mu=\frac{1-\cos\alpha}{\sin\alpha}=\frac{2\sin^2\frac{\alpha}{2}}{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}=tg\frac{\alpha}{2}$$

$tg\frac{\alpha}{2}=\mu=0.58\Rightarrow\frac{\alpha}{2}\approx\frac{\pi}{6};\ \alpha\approx\frac{\pi}{3}$





4.



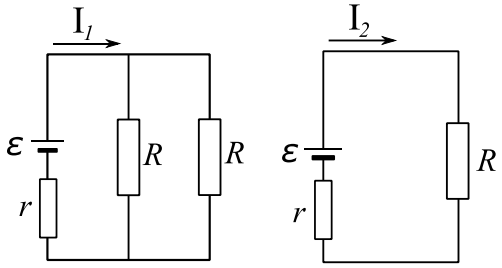
$$\left.\begin{aligned} C_1(t_3 - t_1) &= C_2(t_2 - t_3) \\ C_1(t - t_1) &= 3C_2(t_2 - t) \end{aligned}\right\} \Rightarrow \frac{t_3 - t_1}{t - t_1} = \frac{t_2 - t_3}{3(t_2 - t)}$$

$$t = \frac{t_3(3t_2 - t_1) - 2t_1t_2}{2t_3 - 3t_1 + t_2} = \frac{40 \cdot (3 \cdot 100 - 10) - 2 \cdot 10 \cdot 100}{2 \cdot 40 - 3 \cdot 10 + 100} = \frac{40 \cdot 290 - 2000}{150} = 64^0 C$$

5.

1) $\varepsilon = I_1\left(r + \frac{R}{2}\right)$ (1)

$$Q = I_1^2 \cdot \frac{R}{2} = \frac{\varepsilon^2}{\left(r + \frac{R}{2}\right)^2}\frac{R}{2} \quad (2)$$



2) $\varepsilon = I_2(r + R)$ (3)

$$Q = I_2^2 R = \frac{\varepsilon^2}{(r + R)^2} R \quad (4)$$

Из (2), (4): $\dfrac{\varepsilon^2}{\left(r + \frac{R}{2}\right)^2} \cdot \dfrac{R}{2} = \dfrac{\varepsilon^2}{(r + R)^2} \cdot R$

$\left(r + \frac{R}{2}\right)\sqrt{2} = r + R; \quad r\left(\sqrt{2} - 1\right) = R\left(1 - \frac{1}{\sqrt{2}}\right);$ $R = \sqrt{2} \cdot r = \sqrt{2} \cdot 100 \approx 141$ Ом